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Jet Noise

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1. Introduction

I AM honored that this great association, devoted to the sciences that permit vehicles heavier than air to follow controlled paths away from the earth's surface, has asked me to visit the country of the Wright Brothers, who first achieved this aim, and to speak in their memory. I am for a second time so speaking, having given the Wilbur Wright Memorial Lecture in 1960, on the subject of "Mathematics and Aeronautics," to the Royal Aeronautical Society.[1] I spoke then, with enthusiasm, of the many-sided abilities revealed by the documents of the Wright Brothers' career, a many-sidedness that made them, in a way, typical of the greatest men of this century. More recently, in Michigan, visiting Henry Ford's admirable reconstruction of the Wright Cycle Shop, with its useful wind tunnel in the small back room, I was reminded how their career contains also, for this century, a lesson—that individual initiative in modest surroundings can spark off a transformation of the human condition.

This lecture is also related to yet another that I gave in my own country, the Bakerian Lecture[2] of 1961, to the Royal Society of London, on the subject of "Sound Generated Aerodynamically." In that lecture, to an audience representing almost all the sciences, I attempted to sketch in its broadest features a new science of the middle of this century—that of sound radiation fields that are by-products of airflows. Having given such a broad outline of that new science to a general scientific audience, I am glad to be able to describe in more detail the part of it which I have explored most myself, namely, its application to the problem of the noise of jets. This topic alone is a very complex one and deserves, in its own right, the extended treatment that can be given appropriately to an audience concerned professionally with the cacophony that is, to a greater or lesser extent, inseparable from turbojet and rocket propulsion.

The noise of a jet can be regarded as a separate entity, even when noise emerges also from the exhaust nozzle, arising from conditions in the nozzle or upstream of it. For, whatever those conditions may be, a new, very intense turbulence is created in the shear layer where the the jet fluid, after leaving the nozzle, exchanges momentum with the atmosphere. The noise that this generates can be thought of as, to a large extent, additive to combustion noise or turbomachinery noise. Furthermore, to a good approximation, it is uninfluenced by the presence of the nozzle and can be visualized as sound radiated into free space by a limited extent of turbulent gas flow.

It is simply the radiated sound, representing energy actually extracted from the jet and propagated away through the atmosphere, that I shall discuss. My concern, in other words, will be with the atmosphere pressure fluctuations in the "radiation field" or "far field," where those pressure fluctuations fall off in proportion to the inverse first power of the distance. Within a particular narrow band of frequencies, the far field of any noise source is the region whose distance from it substantially exceeds one wavelength.[2] By contrast, the near field, within a wavelength or so of the source, includes not only outwardly propagating waves but also local reciprocating motions and pressure fluctuations, such as may be induced directly by fluctuating vortex movements in the turbulent gas flow. This near sound field is of concern to the designer of structures to be placed near jet orifices, but the body of knowledge which has been sought after most by aircraft and rocket engineers comprises the acoustic power radiated by a jet, its frequency spectrum, and its directional distribution.

Dr. Michael James Lighthill is the Director of the Royal Aircraft Establishment, the premier establishment of its kind in Europe. Born in January 1924, Dr. Lighthill was educated at Winchester College and at Trinity College, Cambridge University. He joined the Aerodynamics Division of the National Physical Laboratory in 1943. From 1946 to 1950, he was Senior Lecturer in Mathematics at the University of Manchester, where, in 1950, he was named Beyer Professor of Mathematics, a position he held until his present appointment. Elected a Fellow of Trinity College, Cambridge University, in 1945, Dr. Lighthill also was honored with a Fellowship in the Royal Society in 1953. The Royal Aeronautical Society awarded him its Bronze Medal for work leading to advances in Aerodynamics in 1955, "for his contributions to theoretical high speed aerodynamics which recently had included outstanding work on the source of noise, from jets particularly." Dr. Lighthill was made a Foreign Member of the American Academy of Arts and Sciences in 1958. He was elected Fellow of the Royal Aeronautical Society in 1961 and is an honorary Fellow Member of AIAA.

Received April 25, 1963. In essence, this is the Wright Brothers Lecture presented at the 31st Annual Meeting of the Institute of Aerospace Sciences, January 21, 1963. The author is deeply grateful to P. O. A. L. Davies, J. E. Ffowcs Williams, and Alan Powell for their kindness in putting unpublished material at his disposal and drawing his attention to numerous published reports in connection with his preparations for this lecture.

* As follows from integrating the measured fluxes of mean-flow and turbulent-flow energy.

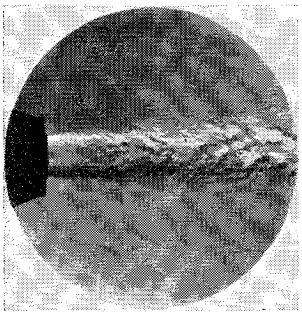


Fig. 1 Subsonic jet

Now, I propose to regard this acoustic power output as a by-product of the main jet flow, not a phenomenon that has itself any marked influence on the flow development. This view is well substantiated for subsonic jets (Fig. 1), which radiate, at most, 10^{-4} of their power as sound, mostly from a region within 4 or 5 diam of the orifice, namely, the intensely turbulent shear layer between the central jet core and the atmosphere. Yet viscous damping of the turbulence in that short length of jet extracts already over a quarter of the normal jet power,* so that evidently the additional damping of less than 0.0001 by sound radiation is negligible by comparison. Admittedly, certain jets at very low Reynolds numbers, which are only marginally unstable, show a sensitive interaction with quite weak sounds,[2] but jets at the much higher Reynolds numbers characteristic of engineering applications are fully turbulent independently of any acoustic stimulation.

For supersonic jets, this distinction is somewhat less clear-cut, since jets can be generated in the laboratory (Fig. 2) at respectable Reynolds numbers (though usually less than 10^6) that show a special kind of instability of acoustic origin. Powell first described and explained this usually asymmetric disintegration (see the Bakerian Lecture,[2] Sec. 4.5 and references there given, together with a recent major study by Davies and Oldfield[3]) in terms of a mechanism involving the pattern of shock waves in a supersonic jet. When an eddy passes through one of the shock waves, an acoustic pulse is emitted with a strong directional peak in the upstream direction. Under favorable conditions this may, on passing the orifice, generate a new eddy that can extract sufficient energy from the shear, before it reaches the shock, to regain the amplitude of the original eddy and so maintain the cycle. The jet then emits an abnormally high quantity of acoustic power, with a spectral peak at a certain characteristic frequency. However, any substantial irregularities created at the orifice were found by Powell to reduce the amplitude gain in the shear layer sufficiently to destroy the effect and to restore the narrow spread characteristic of most practical supersonic jets.

These undisintegrated supersonic jets (Fig. 3), with which alone I shall be concerned in this lecture, have acoustic efficiencies (that is, acoustic power output divided by jet power) rising rapidly from about 10^{-4} for jet speeds equal to the atmospheric speed of sound to 2 or 3 times 10^{-3} for jets of twice that speed (Fig. 4).[5-7] At higher speeds, however,

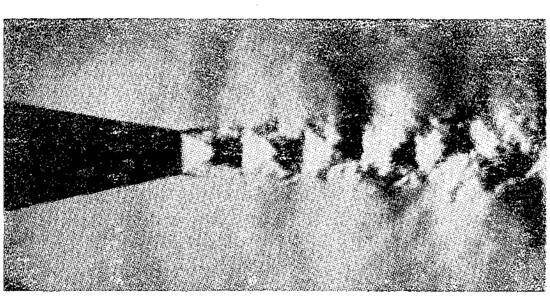


Fig. 2 Disintegrated supersonic jet

including the much higher speeds of rockets exhausts,[8-10] the acoustic efficiency flattens off to a maximum between 5 and 7 times 10^{-3}. These are still values so low in relation to viscous sources of dissipation that the effect of acoustic energy loss on turbulence development is probably negligible. Admittedly, with increasing Mach number, compressibility effects are found to alter the turbulence, lowering its intensity and the consequent rate of spread of the turbulent shear layer (compare Figs. 1 and 3). But the radiation of acoustic energy is too small to account for this effect, which perhaps should be associated, rather, with energy redistributions within the near sound field.

We have, then, a continuous development in acoustic power output of jets, from subsonic jets through the undisintegrated supersonic jets (including turbojet engine exhausts) to jets of speeds many times the atmospheric speed of sound (such as rocket exhausts), and in all of these it remains a minor by-product of the power of the jet. My main purpose in this lecture is to show how one theoretical approach, developed recently by Ffowcs Williams[11] on the basis of ten-year-old work of my own,[12, 13] gives a good understanding not only of the initial rise in acoustic efficiency with jet speed, but also of its subsequent leveling off and of the associated changes in directional distribution and frequency spectrum.

2. Parts of General Aerodynamic Noise Theory Relevant to Jet Noise

The theory begins[2, 12] (as a well-behaved theory should!) with the fundamental equations of motion, expressed in terms of the mass density ρ and the momentum density ρv_i, where v_i stands for the velocity vector (v_1,v_2,v_3). The equations are

$$\frac{\partial\rho}{\partial t} + \sum_{i=1}^{3} \frac{\partial(\rho v_i)}{\partial x_i} = 0 \qquad (1)$$

$$\frac{\partial(\rho v_i)}{\partial t} + \sum_{j=1}^{3} \frac{\partial(\rho v_i v_j + p_{ij})}{\partial x_j} = 0 \qquad (2)$$

and they relate rate of change of mass or momentum in a small volume to the total mass transport or momentum transport out of the volume (ρv_i appears twice because it is both the mass transport density and the momentum density).

The momentum transport appears in two parts: $\rho v_i v_j$ represents direct convection of the momentum component ρv_i by the velocity component v_j, whereas p_{ij} is the stress between adjacent elements of gas, which equally transfers momentum. This stress is the sum of a pure pressure term $p\delta_{ij}$ (where δ_{ij} is 1 when $i = j$ and otherwise is zero) and a viscous stress; in an ideal gas, it represents simply convection of momentum by the motion of the molecules relative to the gas velocity v_i.

Now, one special approximate form for momentum transport yields the classical equations of acoustics; this is a pure isotropic pressure, the variations in which bear to the variation in density a constant ratio a_0^2 (the square of the atmospheric sound speed). If this "acoustic" approximation to the momentum transport is made, Eq. (2) takes the simple form

$$\partial(\rho v_i)/\partial t + a_0^2(\partial\rho/\partial x_i) = 0 \qquad (3)$$

so that, from (1), the density ρ satisfies the classical wave equation

$$\frac{\partial^2\rho}{\partial t^2} - \sum_{i=1}^{3} a_0^2 \frac{\partial^2\rho}{\partial x_i^2} = 0 \qquad (4)$$

However, without making any approximation, one can divide the momentum transport into two parts: first, this "acoustic" approximation, $a_0^2\rho\delta_{ij}$, and, secondly, the remainder, which can be written as

$$T_{ij} = \rho v_i v_j + p_{ij} - a_0^2\rho\delta_{ij} \qquad (5)$$

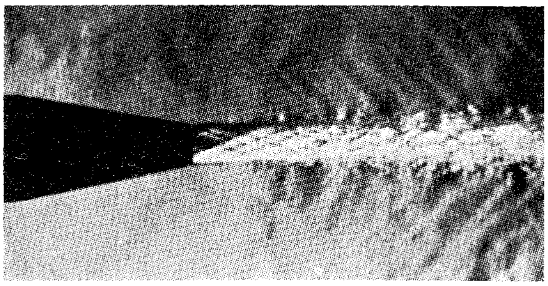


Fig. 3 Undisintegrated supersonic jet

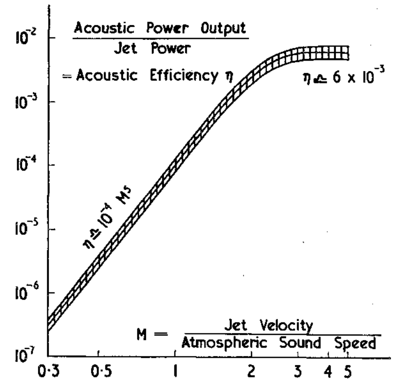


Fig. 4 Acoustic efficiency of jets (excluding disintegrated supersonic jets)

If in the momentum equation the terms involving this part are placed on the right-hand side, one obtains these equations:

$$\frac{\partial \rho}{\partial t} + \sum_{i=1}^{3} \frac{\partial(\rho v_i)}{\partial x_i} = 0 \tag{6}$$

$$\frac{\partial(\rho v_i)}{\partial t} + a_0^2 \frac{\partial \rho}{\partial x_i} = -\sum_{j=1}^{3} \frac{\partial T_{ij}}{\partial x_j} \tag{7}$$

Physically, these state that a fluctuating flow of gas, such as a turbulent jet, generates in the atmosphere outside it the same fluctuations of density as would be produced in a classical *stationary* acoustic medium by a system of externally applied stresses T_{ij}. This view of the sound, as generated in the atmosphere essentially in the manner of a forced oscillation, is suitable, since, as has been shown, the radiated sound is a by-product of the turbulent flow which does not react significantly upon it. Another important advantage is that refraction or scattering of the sound by the flow, as well as its generation, are automatically taken account of in the equivalent applied stress system T_{ij} and do not have to be calculated separately.

On elimination of the momentum density from the Eqs. (6) and (7), one obtains this equation:

$$\frac{\partial^2 \rho}{\partial t^2} - \sum_{i=1}^{3} a_0^2 \frac{\partial^2 \rho}{\partial x_i^2} = \sum_{i=1}^{3}\sum_{j=1}^{3} \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \tag{8}$$

Now, because the forcing term on the right-hand side, that is, the sound source term, is a sum of second derivatives, the atmospheric sound field can be regarded[2, 12] as that due to a continuous distribution of acoustic quadrupoles of strength T_{ij} per unit volume. Physically, one can understand why the quadrupole strength is equal to the stress, by remembering (Fig. 5) that an element of fluid under stress suffers from equal and opposite forces on both sides of it. But any force acting on the medium, in acoustics, is equivalent to a dipole, and so such a pair of equal and opposite forces must be equivalent to two equal and opposite dipoles, that is, to a quadrupole. Elements like T_{11} represent so-called "longitudinal" quadrupoles and elements like T_{12} "lateral" quadrupoles; Fig. 5 shows the directional distribution of pressure amplitude in the radiation field in each case.

Now, the quadrupole nature of the noise sources is important, not only because of the directional distribution of their sound radiation, but also because of its intensity. Although the pressure field of each of the four simple sources composing a quadrupole varies as the inverse first power of the distance from it, differences between inverse first powers of distances from the different points can be neglected in the radiation field because those differences fall off as the inverse square. Hence, acoustic signals arriving at a point in the radiation field from the four simple sources fail to cancel completely only because they must be emitted at different *times* if they are to arrive simultaneously. The amplitude in the radiation field therefore depends entirely on how rapidly the quadrupole strength T_{ij} is changing with time and, indeed, is proportional to the second derivative $\ddot{T}_{ij}$ of T_{ij} with respect to the time t. By contrast, the amplitude at distances from the quadrupole, small compared with an acoustic wavelength is far greater, and it is this property of quadrupole radiation—that local fluctuations are so much in excess of fluctuations in the radiation field—that is the fundamental explanation of the low acoustic efficiency of jets.

The actual solution of Eq. (8) can be written as

$$a_0^2(\rho - \rho_0) = \sum_{i=1}^{3}\sum_{j=1}^{3} \frac{\partial^2}{\partial x_j\, \partial x_i} \int_V \frac{[T_{ij}]}{4\pi r}\, d\tau \tag{9}$$

where V stands for the region of turbulent flow, and r represents the distance from a general point in V to the point (x_1,x_2,x_3) where $a_0^2(\rho - \rho_0)$, which is approximately the pressure fluctuation, is being measured. The brackets around T_{ij} mean that it is evaluated at the retarded time $t - r/a_0$, that is, at the instant when a wave traveling at the speed of sound a_0 had to leave the quadrupole to reach the point of observation at the current time t. The integral in (9) represents what the solution would be if the right-hand side of Eq. (8) for the density were simply T_{ij} itself; and, from this, it evidently follows that the full Eq. (8) has the solution (9). I have said that V stands for the region of turbulent flow; admittedly, it would be more exact to take it as the whole of space, but outside the jet both terms in T_{ij} are small. ($\rho v_i v_j$ falls off like the square of the disturbances, and so does $p_{ij} - a_0^2\rho\delta_{ij}$.) Accordingly, in this formulation it is a good approximation to integrate only over the region of intense turbulent jet flow; indeed, by neglecting contributions from outside it, one is neglecting only how the sound emitted decays by attenuation and finite-amplitude propagation effects (combined), which are better left to be estimated as a separate exercise.

Now, in the radiation field, the differentiations in Eq. (9) can be carried out very simply; no terms due to differentiating the reciprocal of r with respect to x_i or x_j are present, since they fall off at least like $1/r^2$. However, $[T_{ij}]$ represents the value of T_{ij} at a time $t - r/a_0$, which depends on r and hence also on the x_i; indeed, the rate of change of retarded time with x_i is

$$\frac{\partial}{\partial x_i}\left(t - \frac{r}{a_0}\right) = -\frac{x_i}{a_0 r} \tag{10}$$

Using this in (9), one obtains in the radiation field

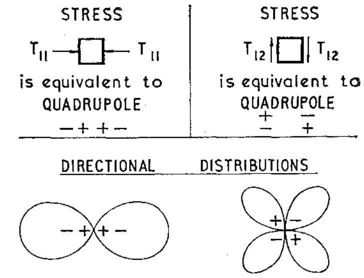


Fig. 5 Equivalence of applied stress to acoustic quadrupole

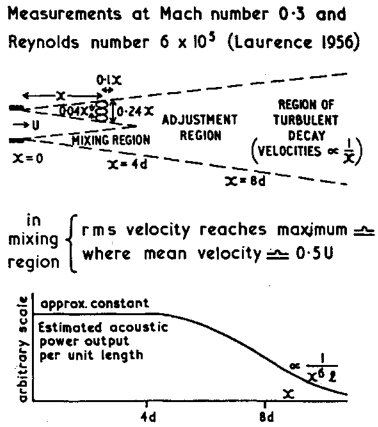


Fig. 6 Stationary cold jet

$$p - p_0 = \sum_{i=1}^{3}\sum_{j=1}^{3} \frac{x_i x_j}{4\pi a_0{}^2 r^3} \int_V [\ddot{T}_{ij}]d\tau \qquad (11)$$

and one recognizes the appearance of the second time derivative $\ddot{T}_{ij}$, resulting in this calculation from the variability of retarded time, a dependence on rapidity of variation of quadrupole strength to be expected from the physical argument I gave earlier.

Equation (11) shows that the coefficients of the characteristic quadrupole terms in the directional distribution of pressure amplitude are obtained by integrating the quadrupole strength T_{ij} over the jet volume V, after double differentiation and evaluation at retarded times. In order to estimate this integral contribution from the whole jet, it is essential to take into account the statistical randomness of turbulent flow.[2, 13] It is a general principle of wave theory that, with well-correlated sources, amplitudes combine linearly, but that, with uncorrelated sources, energy intensities combine linearly. Now, turbulent flow measurements taken simultaneously at nearby points are well correlated, but those at points more remote from one another are almost uncorrelated, and, in the simplest form of the theory, one merely uses this result, neglecting that the $[\ddot{T}_{ij}]$ are not exactly simultaneous values.

This yields a picture of a turbulent flow divided into regions, such that strengths of quadrupoles within any one region are correlated perfectly, but strengths at points in different regions are uncorrelated, the extent l of each independent quadrupole distribution being roughly the size of a typical energy-bearing eddy. For the output of a single region of this kind, of volume V_e,

$$p - p_0 = \sum_{i=1}^{3}\sum_{j=1}^{3} \frac{x_i x_j V_e}{4\pi a_0{}^2 r^3} [\ddot{T}_{ij}] \qquad (12)$$

since within V_e the values of the integrand can be taken equal to their value at the center; on the other hand, the outputs of different regions are perfectly uncorrelated, so that their energy intensities, $<(p - p_0)^2>/\rho_0 a_0$, must be added to give the intensity in the radiation field.

From Eq. (12) one can get a good idea of how the acoustic energy intensity generated by one such eddy at distance r varies with the main parameters involved. It varies approximately as $V_e{}^2\overline{\ddot{T}^2}/\rho_0 a_0{}^5 r^2$, where $\overline{\ddot{T}^2}$ is a typical mean square fluctuation of $\ddot{T}_{ij}$, which also can be written $\omega^4\overline{T^2}$, where ω is a typical radian frequency of fluctuation, and $\overline{T^2}$ is a typical mean square value of the quadrupole strength T_{ij}. This gives the useful approximate formula that the total acoustic power output of one eddy divided by its volume V_e (in other words, the acoustic power output per unit volume of turbulence) is proportional to

$$V_e \omega^4 \overline{T^2}/\rho_0 a_0{}^5 \qquad (13)$$

Strictly speaking, this estimation should be done separately for different parts of the turbulent frequency spectrum, but in practice the variation of the product $V_e\omega^4$ over the spectrum is rather small, because high frequencies go with small eddy volumes.

Evidently, an essential assumption in this estimation was that variations of retarded time within an eddy of size l could be neglected; this, for fluctuations at radian frequency ω, would require $\omega l/a_0$ to be small, so that the eddy size l is small compared with the wavelength of the sound it generates. This assumption, that $\omega l/a_0$ be small, is satisfied at low Mach numbers, since the product ωl of a characteristic frequency and eddy size is a characteristic velocity. Furthermore, I shall show that the Mach number range within which the assumption applies can be extended at least to unity by a modification insuring that ω is a frequency in a frame of reference moving with the eddies, for then ωl is approximately a root-mean-square velocity fluctuation, which is a mere fraction of the jet velocity.

3. Turbulent Jet Aerodynamics

Before I can go any further, however, I must describe the main aerodynamic characteristics of subsonic turbulent jets at high Reynolds numbers (Fig. 6).[14–16] The shear layer, or mixing region between the jet and the atmosphere, becomes fully turbulent within half a nozzle diameter of the orifice. Over a distance of 4 or 5 diam, this region grows linearly with the distance x from the nozzle, its width being between 0.2 and 0.25 times x. At any section of the mixing region, the root-mean-square velocity fluctuation reaches a maximum of about $0.14U$, where U is the jet velocity. The correlation of velocity at one point with velocity at a point downstream of it falls off with distance downstream from 1 to 0 in such a way that the area under the curve, the so-called correlation radius of an eddy, is $0.1x$ or a little more. The correlation radius in the lateral direction, however, is only about $0.04x$, and so one must think of eddies as elongated, as a result of being stretched out by the mean shear.

This uniform growth of the fully turbulent region ceases soon after $x = 4d$, where it reaches the axis, and the jet enters a region of adjustment, beyond which, from about $x = 8d$, it evolves differently, with both mean and rms velocities falling off like $1/x$.

Before I go any further, I shall quickly use these experimental data to infer how the fraction $V_e\omega^4\overline{T^2}/\rho_0 a_0{}^5$, which should be proportional to the acoustic power output per unit volume of a low-speed jet, varies with position.[17, 18] In the mixing region, the fluctuations of terms like $\rho v_i v_j$ in the quadrupole strength T_{ij} are in proportion to $\rho_0 U^2$, changing little with x, but the typical frequency ω falls away like U/x, whereas the typical eddy volume V_e grows like x^3. One therefore should expect the power output to vary as $\rho_0 U^8/a_0{}^5 x$ per unit volume of mixing region. But since the volume per unit length increases with x, the sound emitted per unit length of jet should remain constant in this region, the total up to $x = 4d$ being proportional to

$$\rho_0 U^8 d^2/a_0{}^5 \qquad (14)$$

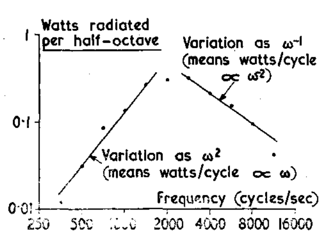


Fig. 7 Spectrum of total acoustic power output from an air jet of diameter 3.9 cm and velocity 290 m/sec (Fitzpatrick and Lee[19])

Beyond $x = 4d$, similar dimensional considerations apply, particularly to the total sound emitted, although, as the fluctuations in T_{ij} fall off, so does the sound emitted per unit length. Indeed, beyond $x = 8d$ where velocities fall off like x^{-1}, the sound emission per unit length would be expected to fall off like $x^{-6}l^{-1}$, where the correlation radius l, though proportional to x for large enough x, varies rather little up to $x = 20d$. This suggests a distribution of sound emission per unit length as in the lower half of Fig. 6, with about half the total coming from the mixing region.

Different parts of the jet emit sound in different frequency bands; in particular, in the mixing region, where a typical frequency $\omega \propto x^{-1}$, the sound emitted uniformly in x should have a frequency spectrum per cycle like ω^{-2}, since $dx \propto \omega^{-2}d\omega$. In the region $x > 8d$, one gets a spectrum varying as as ω^2 or ω^4 accordingly as l increases like x or remains constant. Experimental spectra (see, for example, Fig. 7) are consistent with the view that about half the sound comes from the mixing region and has the ω^{-2} type of spectrum, whereas most of the rest has a spectrum most like ω than like ω^4 and probably comes from the adjustment region.

Experiments agree well, also, as will be shown, with the prediction that acoustic power output should be proportional to $\rho_0 U^8 d^2/a_0^5$, although I must emphasize that I so far have indicated how it can be derived only for low Mach numbers. To understand why it is also a good approximation at higher

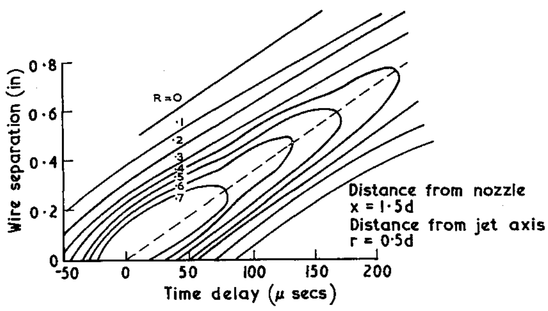


Fig. 8 Curves of constant correlation (Davies, Fisher, and Barratt[20]) between velocities measured at two wires, one directly downstream of the other, in a 1-in.-diam jet at $M = 0.45$

Mach numbers, or to predict directional distributions or peak frequencies, one must, as I mentioned earlier, consider the turbulent fluctuations in a frame of reference moving with the eddies.

In my paper[13] of 1954, I assumed, without conclusive experimental evidence, that at each point of the jet there would be such a frame of reference, roughly speaking "moving with the eddies," in which fluctuations would be considerably slower than in a fixed frame. In other words, time variation at a fixed point would give a misleading impression of high frequency, reflecting merely the speed of convection of the random space pattern past the point, rather than the speed of changes in that pattern as it is swept downstream.

In the past two years this assumption has been given detailed experimental verification, particularly in the work of Davies, Fisher, and Barratt of the University of Southampton[20] (but see their later paper[33] for corrected numerical values of certain expressions calculated from their data). They carried out a systematic series of hot-wire measurements in the mixing region of a jet, using two wires, one placed directly downstream of the other, the distance between them being variable. By correlating simultaneous values of velocity at the two wires, they evaluated the longitudinal correlation radius of an eddy, obtaining good agreement with previous investigators. However, they also correlated the velocity at the upstream point with the velocity observed a short time later at the downstream point, by incorporating a small and

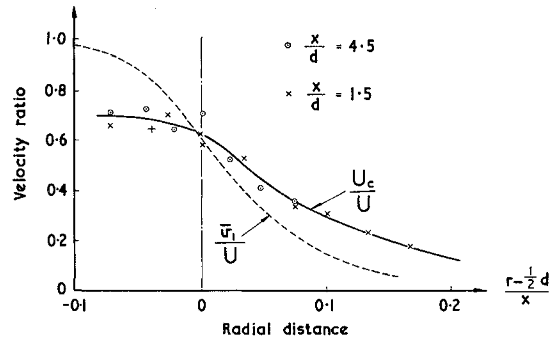


Fig. 9 Radial distribution of mean velocity $\bar{v}$, and of convection velocity U_c in the mixing region of a jet of velocity U (Davies, Fisher, and Barratt[20])

variable time delay in the second hot-wire circuit. By these means they were able to infer how, in a frame of reference moving at any chosen speed, equal to hot-wire separation divided by time delay, the correlation between one turbulent-velocity value and a later one falls off with time.

Their results are plotted best as curves of constant correlation (Fig. 8), that is, curves in a diagram of wire separation against time delay on which the correlation between the measured velocities takes constant values. These show that the frame of reference in which the correlation falls off most slowly with time is rather well defined, and that the rate of fall in this moving frame is only about a quarter of that measured in a fixed frame. This implies that typical frequencies in the frame moving at the eddy convection velocity U_c are about a quarter of those that would be measured at a fixed point.

Davies, Fisher, and Barratt[20] made similar measurements at several stations in the mixing region and at various jet velocities. They found that the velocity of eddy convection varies across the mixing region (Fig. 9) but not nearly so widely as does the mean velocity in the jet; values of U_c from $0.2U$ to $0.7U$ were observed, an average value being $0.5U$. As a measure of rates of change in a frame moving with the eddies, they used the time scale τ, defined as the time taken for the correlation in that frame to fall to a value $1/e$; the reciprocal of τ would be a typical radian frequency of turbulent fluctuations in that frame. By plotting the time scale τ against the reciprocal of the local mean shear (Fig. 10), they discovered the interesting approximate relationship

$$\tau \simeq 4\tfrac{1}{2} \text{ mean shear} \tag{15}$$

at each point of the jet. This means that correlation falls to the value $1/e$ in the time taken for a sphere of fluid to be stretched by the mean shear into an ellipsoid with axes in the ratios $1:4\frac{1}{2}:20$.

Davies, Fisher, and Barratt[20] also obtain a number of other

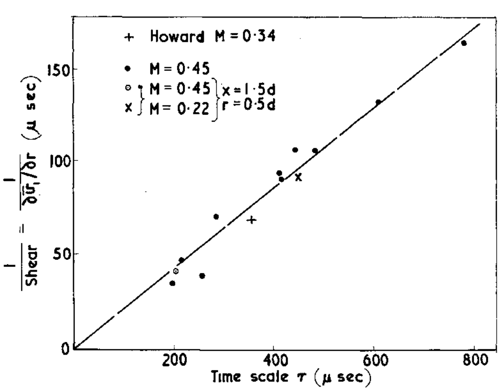


Fig. 10 Variation of time scale with local mean shear (Davies, Fisher, and Barratt[20])

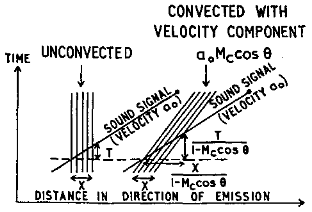


Fig. 11 Increase of emitting volume and emission time differences, because of convection, by the factor $1/(1 - M_c \cos\theta)$

results of importance to turbulent-shear-flow theory as well as to jet-noise theory. These include the following relationships between the rms velocity $v_1' = \{\langle(v_1 - \bar{v}_1)^2\rangle\}^{1/2}$, the longitudinal correlation radius l, the time scale τ, and the mean shear:

$$v_1' \simeq 0.2(\partial\bar{v}_1/\partial x_2)l \tag{16}$$

$$v_1'\tau \simeq 0.9l \tag{17}$$

or, if $\omega = 1/\tau$, then $\omega l \simeq 1.1v_1'$. The latter shows that a typical radian frequency $\omega = 1/\tau$ in a frame moving with the eddies is approximately the rms velocity divided by the length scale and is at most about a sixth of the corresponding frequency in a fixed frame, which is the mean velocity divided by the length scale.

4. Subsonic Jet Noise Theory

After this description of jet aerodynamics, I can return now to the sound radiation field and explain why, for calculating it, the turbulent eddies must be viewed as a pattern of quadrupoles which is moving downstream.[2,13] I hinted earlier at one reason for this, namely, that, by confining consideration to frequencies in a frame of reference moving with the eddies, the range in which $\omega l/a_0$ is small is extended up to at least $M = 1$. This is necessary in order to justify the neglect of differences in retarded times within an eddy, and the results I have just quoted on frequency in the moving frame show that in a subsonic jet $\omega l/a_0$ is indeed everywhere less than one-sixth.

A more basic reason, and one that applies even at low jet Mach numbers, why in estimating

$$\int_V \ddot{T}_{ij} d\tau$$

over an eddy the time rate of change must be taken in a frame of reference moving with the convection velocity, is as follows. If instead one were to inflate that time rate of change by adding to it a space rate of change times a convection velocity, the extra contribution to the volume integral from the space rate of change could be transformed at once into a surface integral over a surface outside the eddy. Its contribution to the intensity of acoustic radiation would then be zero, because values on that surface would be uncorrelated with values within the eddy; so, in the end, one would be left simply with the time rate of change in the moving frame.

These considerations indicate that the eddy-sized regions into which we earlier imagined the jet as divided must be thought of as moving quadrupole sources of sound. But accoring to well-established theory (see the Bakerian Lecture[2] for full references), a moving volume V_e in which the quadrupole strength per unit volume is T_{ij} has the sound radiation field

$$p - p_0 = \sum_{i=1}^{3}\sum_{j=1}^{3} \frac{x_i x_j V_e[\ddot{T}_{ij}]}{4\pi a_0^2 r^3(1 - M_c \cos\theta)^3} \tag{18}$$

where M_c is the velocity of convection U_c divided by the atmospheric sound speed a_0. The effect of convection appears merely in the additional $(1 - M_c \cos\theta)^{-3}$ factor, in which θ is the angle between the direction of emission and the jet direction.

This factor, which increases the sound emitted forward substantially more than it decreases that emitted backward, can be understood from a diagram (Fig. 11) in which time is plotted vertically and distance in the direction of emission horizontally; the figure shows a sound signal being emitted with velocity a_0 from unconvected eddies and from eddies whose convection velocity component in this direction is $a_0 M_c \cos\theta$. It shows that, for a given quadrupole strength per unit volume, the emission is increased when this velocity component is positive, because the total emitting volume is increased by the factor $1/(1 - M_c \cos\theta)$. Furthermore, because the radiation field of a quadrupole is due entirely to incomplete canceling of signals from positive and negative sources, owing to differences in times of emission, two extra factors $1/(1 - M_c \cos\theta)$ appear, representing the increases in those time differences. These are related closely to the Doppler increase in the observed frequency.

Now, this incorporation of a directional factor $(1 - M_c \cos\theta)^{-3}$, in the pressure amplitude generated by each eddy in its radiation field, calls for a directional factor $(1 - M_c \cos\theta)^{-6}$ in the intensity of sound generated by the eddy. When one considers the sound intensity generated by the jet as a whole, however, or even by a given restricted extent of jet, this directional factor is modified, as Ffowcs Williams first showed.[21] Figure 12 depicts the moving eddies within a given extent of jet X and shows that this extent must be multiplied by $(1 - M_c \cos\theta)$ to include only that number whose radiation arrives simultaneously. It follows (multiplying this by the directional factor on intensity) that the acoustic intensity field per unit volume of jet takes the form

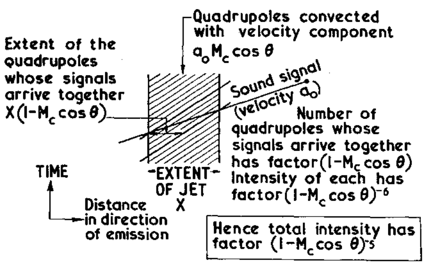


Fig. 12 Stationary jet; acoustic intensity field per unit volume of jet is

$$\frac{\langle(p-p_0)^2\rangle}{\rho_0 a_0} = \frac{V_e}{\rho_0 a_0}\left\langle\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{x_i x_j[\ddot{T}_{ij}]}{4\pi a_0^2 r^3}\right)^2\right\rangle(1-M_c\cos\theta)^{-5}$$

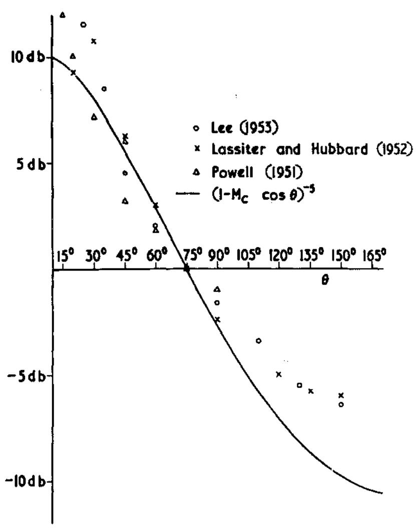


Fig. 13 Directional distribution of sound radiation from air jets at 300 m/sec, in decibels relative to $\theta = 75°$

$$\frac{\langle(p - p_0)^2\rangle}{\rho_0 a_0} = \frac{V_e}{\rho_0 a_0}\left\langle\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{x_i x_j[\ddot{T}_{ij}]}{4\pi a_0^2 r^3}\right)^2\right\rangle \times (1 - M_c \cos\theta)^{-5} \quad (19)$$

where the exponent 5 replaces the value 6 given in my paper[13] of 1954. Ffowcs Williams[21] has made a similar modification for a jet in flight which there is not time to discuss here.

It is very satisfactory that, by incorporating this simple directional factor, a basically low Mach number theory can be made valid for jets at speeds at least up to the atmospheric sound speed a_0. The reason, as has been shown, is that, when eddies are viewed in a moving frame, the assumption at the basis of the approximations becomes that the rms turbulent velocity, not the mean velocity, is small compared with a_0. It is, however, still more satisfactory that this factor explains fully what is one of the most noticeable features of jet noise, namely, its marked directionality, that is, greater intensity for small angles θ between the jet direction and the direction of emission, a variation that becomes more pronounced as the jet velocity increases. For example, in Fig. 13, showing the directional distribution of sound for stationary cold jets at 300 m/sec, the curve represents the $(1 - M_c \cos\theta)^{-5}$ factor, using an average velocity of convection equal to half the jet velocity. The measured directional distributions are in generally good agreement; they fall off somewhat slower behind the orifice than the formula would predict, probably because other sources of sound are additionally detectable in this region. (These include the sound fields of the weaker peripheral eddies with lower velocities of convection and also, in practical experiments, some reverberation of the much higher intensity sound transmitted forwards.) I shall show later that not only the directional distribution at a given jet velocity but also the measured change in directional distribution with jet velocity are in good accord with theory.

Indeed, theory indicates only how directional distribution should *change* with jet velocity, through changes in the $(1 - M_c \cos\theta)^{-5}$ factor, while leaving open the possibility that, superimposed on this factor, there may be some preferred orientation of the quadrupoles T_{ij}. Experimentally, the total radiated sound does not exhibit any marked effect of this kind, but the high-frequency sound does; to explain this, I argued in my paper[13] of 1954 that, in any region of large mean shear, there would be some predominance of lateral quadrupoles with directional peaks at 45° to the direction of flow. This followed from an expression for $\dot{T}_{ij}$ as the product of rate-of-strain and pressure, plus other terms whose effect should be smaller. This relation would lead one to expect a dominant lateral quadrupole associated with the mean rate-of-strain, with directional peaks at 45° to the flow direction, combined with randomly oriented quadrupoles (associated with fluctuating rates-of-strain and other terms in $\dot{T}_{ij}$).

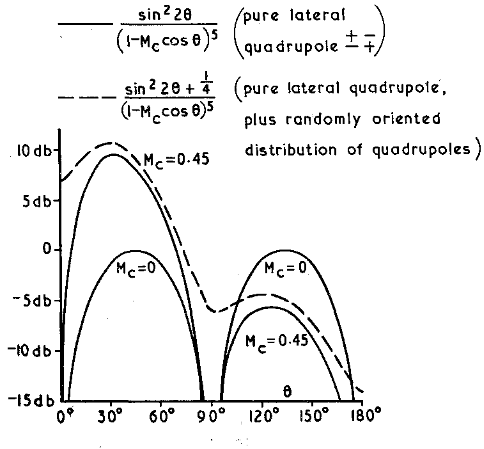


Fig. 14 Directional distributions of intensity in decibels

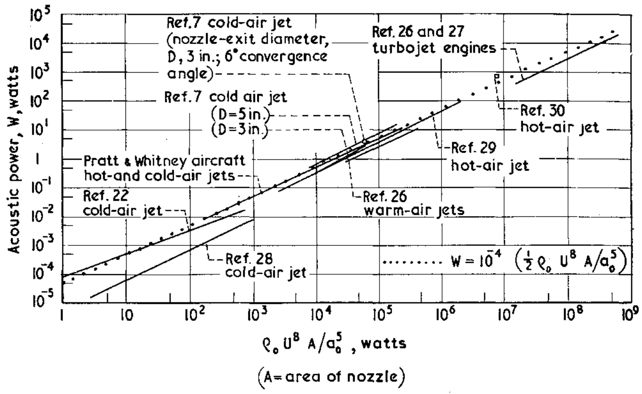


Fig. 15 Acoustic power radiated by jets at speeds up to 600 m/sec

Figure 14 shows the type of directional distribution that would be expected, with a peak around 30° and a subsidiary peak around 120°, and it is significant that the upper-frequency part of the jet noise spectrum, which is regarded as emitted by the region of high mean shear, does have this type of directional distribution.[5, 24, 25]

I come now to the rate of increase of acoustic intensity with jet velocity, which is expected to be in proportion to the product of the factor $V_e\omega^4\overline{T^2}/\rho_0 a_0^5$ (per unit volume) which I discussed earlier and the directional factor $(1 - M_c \cos\theta)^{-5}$. At low Mach numbers the former factor should, as I explained, increase as the eighth power of the jet velocity U. However, turbulence measurements at the higher subsonic Mach numbers[14, 15] indicate a definite reduction in turbulent intensity, that is, in the ratio of rms velocity to mean velocity, as the Mach number increases. With this goes a small reduction in rate of spread of the mixing region (which, being much easier to measure, is valuable collateral evidence for the intensity reduction), preparatory to the much greater reduction in rate of spread that occurs, as I mentioned earlier, at still higher Mach numbers (compare Figs. 1 and 3). The work of Davies, Fisher, and Barratt[20] suggests that a reduction in rms velocity also should reduce the product ωl of the eddy size l and the frequency ω in a moving frame and so reduce also the product $V_e\omega^4$. If the rms velocity goes up as the $\frac{3}{4}$th power of the jet velocity U, as the limited evidence at the higher subsonic jet speeds seems to indicate, then $V_e\omega^4\overline{T^2}$ should be proportional to U^6, rather than to U^8 as it is at low Mach numbers.

Now, this checks well with measurements of acoustic intensity at $\theta = 90°$, where the directional factor $(1 - M_c \cdot \cos\theta)^{-5}$ has no effect because its value is unity; such measurements[5, 25] show close proportionality to U^6 between jet Mach numbers of 0.5 and 1. Furthermore, measurements at other angular positions show dependence on U equally consistent with the idea of a basic U^6 term, modified by the directional factor $(1 - M_c \cos\theta)^{-5}$. For example, at $\theta = 20°$, measurements of acoustic intensity[5, 25] are roughly proportional to U^9 between $M = 0.5$ and 1, where the factor $(1 - M_c \cos\theta)^{-3}$ does vary in close proportion to U^3. Similarly, the integral of $(1 - M_c \cos\theta)^{-5}$ over a sphere varies in close proportion to U^2, which superimposed on the U^6 variation at $\theta = 90°$ gives a U^8 variation for the total power output, as is observed. A small selection of the numerous data bearing on this eighth-power law for total power output is included in Fig. 15, taken from a report[7] of the NASA Lewis Research Center.

People always find it difficult, at first, to believe that the fact that the total acoustic power output of jets varies in the high subsonic region as U^8, which happens to be the same power as is given by low Mach number theory, is because of the canceling out of two corrections to that theory. However, both corrections are indisputably necessary; the directional factor is supported excellently by experiment, as well as being theoretically essential, and the decrease in turbulent intensity with Mach number is also well substantiated.

5. Supersonic Jet Noise Theory

I must pass now to the case of jet Mach numbers greater than 1, meaning always by jet Mach number the one relevant to jet noise, namely, the jet velocity divided by the atmospheric sound speed. It has been clear for many years that, to deal with this case, a major modification to the theory would be needed, namely, abandonment of the concept of neglecting variation of retarded time within an eddy, for it has been shown that the fraction $\omega l/a_0$, which must be small to justify this, becomes almost $\frac{1}{6}$ at a Mach number of 1, even when ω is the radian frequency in the moving frame of reference. Furthermore, the continued validity of the directional factor $(1 - M_c \cos\theta)^{-5}$, derived from neglecting variation of retarded time within an eddy, is clearly suspect when that factor becomes large or infinite. This requires only that values near 1, or greater, are taken by the Mach number of convection M_c, whose measured values (as was shown) lie between $0.2M$ and $0.7M$. This indicates that the neglect of variations in retarded time must be abandoned when the jet speed exceeds to any marked extent the atmospheric sound speed.

Fortunately, Ffowcs Williams has shown during the past year[11] how, with some increase in mathematical complication, the theory can be worked through without this restrictive approximation, and he has obtained results in good agreement with observations on supersonic jets. His work confirms that, when the Mach number of convection M_c is supersonic, the sound intensity has a pronounced directional peak where $\cos\theta = 1/M_c$, but a finite one instead of the infinite one that the $(1 - M_c \cos\theta)^{-5}$ factor would suggest. The predominance of radiation near the angle for which $\cos\theta = 1/M_c$ is, indeed, clearly visible on Schlieren photographs (Fig. 3) and can be simple-mindedly viewed as a distribution of ballistic shock waves generated by the motion of eddies through the atmosphere faster than the atmospheric speed of sound.

To get a broad understanding of Ffowcs Williams' calculations, it is sufficient to appreciate how he derives a finite value for the intensity at this directional peak, particularly since that value is of major importance in determining the total acoustic power output. The easiest way to such appreciation starts from Fig. 11, which showed an eddy in a diagram with time plotted vertically and distance in the direction of emission horizontally, in order to explain the occurrence of the factors $1/(1 - M_c \cos\theta)$. If this diagram is used in the case

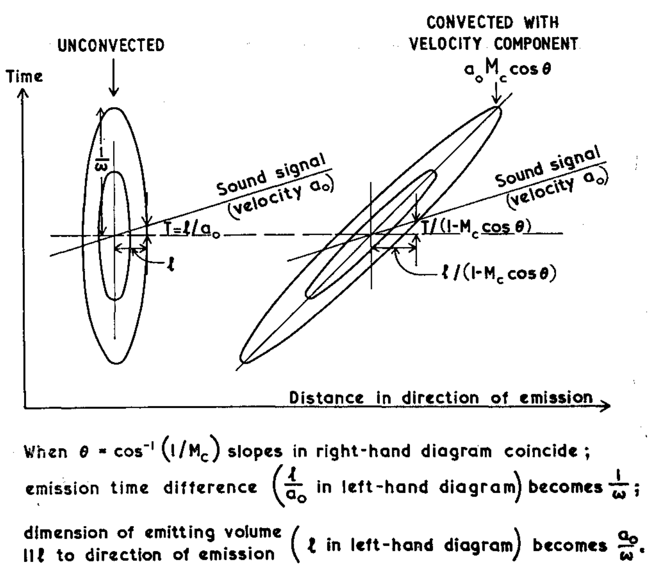


Fig. 16 Illustrating the Ffowcs Williams theory of supersonic eddy convection

when $M_c \cos\theta = 1$ (so that the slope of the sound signal becomes equal to the slope of the lines signifying the motion of the eddy), it appears inevitable that the emitting volume and emission time differences increase to infinity. However, from the experimental work of Davies, Fisher, and Barratt,[20] one knows that this picture of a moving eddy is unrealistic, because even in the moving frame of reference it is changing gradually, and velocity values are losing correlation with earlier values. Figure 8 showed this clearly and suggests the revised version of Fig. 11 which is shown as Fig. 16, still with time plotted vertically and distance in the direction of emission horizontally.

This represents properly the decay of an eddy with time, by means of curves of constant correlation or, more precisely (taking into account the three-dimensional character of the correlation function), curves on which its surface integral over planes at right angles to the direction of emission remains constant. In the case when the component $a_0 M_c \cos\theta$ of the convection velocity in that direction is considerably less than the atmospheric sound speed a_0, one sees that the emitting volume and emission time differences are still to a good approximation increased by the factor $1/(1 - M_c \cos\theta)$. However, in the case when the two speeds coincide, neither the volume nor the time difference becomes infinite; in particular, the emission time difference, which was formerly l/a_0, becomes $1/\omega$, where ω is a typical radian frequency in the moving frame. As for the emitting volume, its dimensions perpendicular to the direction of emission do not change, but its dimension parallel to the direction of emission increases from l to a_0/ω. Thus, at the angular position $\theta = \cos^{-1}(1/M_c)$, both the emitting volume and the emission time differences increase by the factor $a_0/\omega l$ instead of by the factor $1/(1 - M_c \cdot \cos\theta)$. The deduction of the acoustic radiation field from this goes through exactly as before, with $1 - M_c \cos\theta$ replaced throughout by $\omega l/a_0$, and, in particular, one finds $(\omega l/a_0)^{-5}$ appearing in place of $(1 - M_c \cos\theta)^{-5}$ in the expression for acoustic intensity in the direction $\theta = \cos^{-1}(1/M_c)$.

Ffowcs Williams[11] has carried out the calculation for a more general case, when the two slopes do not coincide but may be near each other in value, and he finds that $(1 - M_c \cos\theta)$ in this more general case should be replaced by

$$\{(1 - M_c \cos\theta)^2 + (\omega l/a_0)^2\}^{1/2} \tag{20}$$

In particular, the acoustic intensity field of unit volume of turbulent jet is proportional to

$$\frac{\langle (p - p_0)^2 \rangle}{\rho_0 a_0^5} = \frac{V_e \omega^4 \overline{T^2}}{\rho_0 a_0} \frac{1}{4\pi r^2} \times \left\{(1 - M_c \cos\theta)^2 + \left(\frac{\omega l}{a_0}\right)^2\right\}^{-5/2} \tag{21}$$

if one neglects for simplicity the additional directional effects arising from any preferred quadrupole orientations.

In estimating the terms in this, it is fair to assume as a basic property of turbulence what Davies, Fisher, and Barratt found in their jets, namely [see Eq. (17)], that the product ωl of frequency and length-scale in a frame of reference moving with the eddies is approximately the rms turbulent velocity. Then both the peak intensity and the total power output given by Eq. (21) must vary as U^3, once the Mach number of convection is well supersonic, say for jet speeds three or more times the atmospheric sound speed. Since the jet power also varies as U^3, this means that the acoustic efficiency, that is, the ratio of acoustic power output to jet power, becomes constant above about this speed, as I indicated earlier to have been found experimentally.

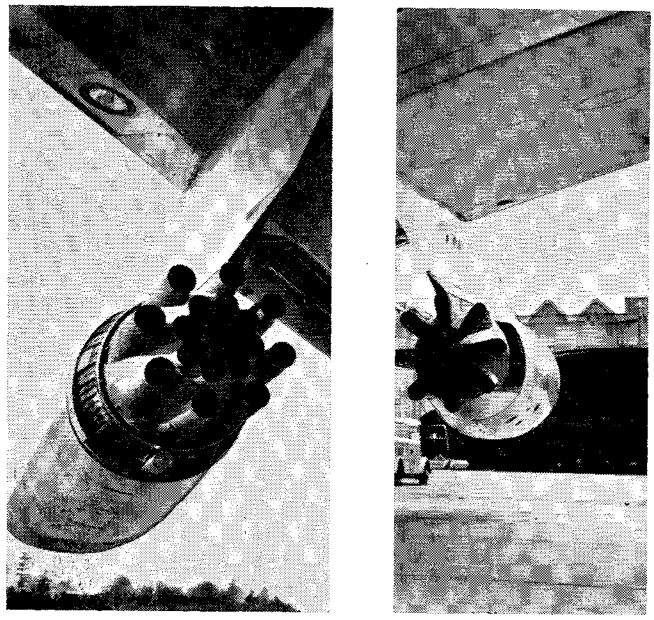
For subsonic jets, it has been shown that acoustic power output varies as U^8, so that acoustic efficiency varies as U^5, experimental values being close to $10^{-4}(U/a_0)^5$ for subsonic jets (Fig. 4). In the upper half of the subsonic range, it was shown that the correctness of this law was maintained by two opposing factors: the gradual reduction in turbulent intensity with increasing Mach number and the substantial increase in forward emission with increase in M_c, the Mach number of convection. It is now clear that this latter effect becomes weakened as M_c increases further, owing to the new term $(\omega l/a_4)^2$ in the denominator of (21). It fails therefore to outrun the reduction in turbulent intensity but rather begins, as M_c approaches 1 (that is, for jet velocities approaching twice the atmospheric sound speed), to conspire with it. The rate of rise of acoustic efficiency then falls rapidly to zero, and the efficiency levels out at a constant (experimentally about 0.006) at jet speeds around three times the atmospheric sound speed.

One consequence of this for rocket noise is that a very great length of rocket exhaust will be radiating sound at approximately constant acoustic efficiency, that is, in direct proportion to the jet energy dissipated. This length should extend to where the jet velocity has fallen below three times the atmospheric sound speed, in other words, to where the jet gases themselves are moving subsonically. This conclusion, that the noise sources extend over a substantial length of rocket exhaust, perhaps as much as 30 nozzle diameters, is in good agreement with observation.[9]

After this reasoned account of the changes, throughout the Mach number range, in acoustic efficiency and directional distribution, I should like to point out that the theory seems to give a resonable explanation also of changes in peak frequency with jet velocity. At low Mach numbers, a typical radian frequency ω for a moving eddy in the mixing region is measured by Davies, Fisher, and Barratt[20] as 1.3 U/x at a distance x from the nozzle. This rises to 0.3 U/d where the peak frequency is expected to be emitted, that is, at the end of the mixing region. On the other hand, this frequency typical of the main energy-containing eddies may not be quite so large as the frequency of the eddies that make $V_c\omega^4\overline{T^2}$ a maximum. The eddy volume V_e (proportional to an eddy dimension cubed) and the frequency factor ω^4 pull different ways in this expression, but the frequency factor (carrying the higher exponent) must be expected to win and to cause the peak frequency to be a multiple greater than 1 of the typical eddy frequency 0.3 U/d. Experimentally, the peaks are somewhat flat, as I have already shown, but they occur at a frequency usually between 1 and 1.5 times U/d, which is consistent with the argument I have just given.

Next, as the jet velocity increases, two opposing influences come into play, of which one is the Doppler factor $1/(1 - M_c \cdot \cos\theta)$ on the peak sound frequency, tending to make it rise more rapidly than in proportion to U. The other opposing influence is that the typical frequency ω of the eddies themselves at the end of the mixing region tends to rise less rapidly than in proportion to U. For it has been shown that ωl is proportional to the rms turbulent velocity, which itself increases less rapidly than U. Furthermore, as U increases, the length of the mixing region increases, and so stretching in the shear layer gives the longitudinal dimension l of an eddy at the end of it an opportunity to become greater. Both these factors reduce ω and may be expected to outweigh the Doppler factor $1/(1 - M_c \cos\theta)$ in the radiated frequency. Experimentally,[5, 7, 19, 22–25] one finds that the peak frequency does increase less than in proportion to jet speed as the latter approaches the speed of sound.

Finally, for very high speeds, the directional peak of the sound emission by eddies of frequency ω should have a frequency equal to ω multiplied not by the Doppler factor $1/(1 - M_c \cos\theta)$ but by the corrected factor amplifying emission time differences, that is, $a_0/\omega l$. If one assumes again that the eddies most effective for sound emission have a frequency about three times as great, then the peak frequency should be $3a_0/l$, with l equal to the longitudinal length-scale at the end of the mixing region. Experimentally,[8–10] the peak frequency of the sound radiation field of a rocket exhaust is about a_0/d, but it is not impossible that in 30 diam or so the longitudinal length-scale l may have risen as high as $3d$, which would explain the observed figure.



a) b)

Fig. 17 Noise suppressors on the Boeing 707; a) with Pratt & Whitney 3C-6 engine, b) with Rolls Royce Conway engine

6. Conclusion

To sum up, the theory gives a reasonably clear explanation of the sound radiation fields of jets in terms of observations on the turbulence in those jets (although not pretending to deal with other noise sources that may be present along with the jet or with losses associated with propagation of the jet noise). In particular, the observed variations in acoustic efficiency, directional distribution, and frequency spectrum, with increasing jet velocity, are accounted for adequately by the representation of a jet as a distribution of acoustic quadrupoles in an otherwise undisturbed atmosphere, the quadrupole strengths being well correlated only within convected eddies of limited extent and limited lifetime.

To go beyond the matter of physical understanding, and inquire concerning means for reducing the noise, elicits from the theory a somewhat pessimistic answer, namely, that the one parameter that affects total acoustic power output to any appreciable extent is rms turbulent velocity, which in turn depends principally on jet speed. By far the best way to reduce noise output is, therefore, to reduce jet speed; and, indeed, if this is already less than twice the atmospheric sound speed, the factor by which noise output is reduced is the sixth power of the factor by which jet speed is reduced, for a constant value of the jet thrust. This has been one of the motive forces behind the trend to engines of high bypass ratio.

Apart from this, only two successful methods of reducing jet noise radiation for a modest weight penalty (with no reduction in jet speed) have been found (see the Bakerian Lecture[2] for full references). One of these contrives to reduce rms turbulent velocity by diminishing the relative velocity of the jet and the air adjacent to it, as a result of nozzle shaping that induces forward motion of much of that adjacent air. The other method makes use of the marked directionality of jet noise, from which it follows that the peak noise from a cluster of nozzles will be less than the sum of the peak noise from each separately. This is because noise in the peak direction from each nozzle will be redirected in some different direction on encountering one of the others. Several good designs of noise suppressors for aircraft (see, for example, Fig. 17) have managed to make use of both these principles, and reductions of up to 10 db (in peak sound) can be made with an overall performance loss, which, although severe, is not crippling economically. In the long run, however, to bring down the jet velocity is the only satisfactory solution. In contrast with all this, hopes of reducing the noise of rockets, at any rate after they are airborne, when even moderate penalties on thrust and weight are likely to be quite unacceptable, appear very slender indeed.

I have tried in this lecture to build up a picture, by a combination of theoretical analysis and experimental evidence, of how the shearing motions in a turbulent jet manage to shed some of their energy as sound radiation. I hope that a few of the ideas that I have presented may prove useful to those who either may be faced with practical jet-noise problems or who have in mind experiments to throw new light on this important and difficult subject.

Appendix A: Influence of the Ratio of Jet Density to Atmospheric Density

A matter, not discussed in the lecture, on which experimental data have not reached complete agreement is the effect of differences between jet density ρ_J (defined as the average density across the orifice) and atmospheric density ρ_0. Some good correlations of power output with $\rho_0 U^8 A/a_0^5$, such as Fig. 15, have been obtained.[7, 32] However, other work,[5, 6, 23] using values of ρ_J considerably smaller than ρ_0, has demonstrated a resulting reduction in the acoustic power.

This might at first sight, through its dependence on $\overline{T^2}/\rho_0$ [Eq. (13)], be expected to vary as ρ_J^2/ρ_0 rather than as ρ_0. However, the peak sound is believed to originate in the center of the mixing region, where a typical density ρ_I should be intermediate between ρ_0 and ρ_J, suggesting dependence on ρ_I^2/ρ_0, which does not fall off with decreasing ρ_J so rapidly as ρ_J^2/ρ_0. The situation is complicated further when, as often happens, the velocity of sound a_J in the jet differs considerably from a_0. Then the term $p_{ij} - a_0^2\rho\delta_{ij}$ in T_{ij} [Eq. (5)] represents sound generation approximately equivalent [by arguments like those leading to (B3) below] to that from a source distribution of strength $(1 - a_0^2/a_J^2)\partial^2 p/\partial t^2$ per unit volume. Our new knowledge (Sec. 3) on frequencies in the mixing region in a frame moving with the eddies indicates that, when a_J^2/a_0^2 is large (which generally goes with small ρ_J/ρ_0), this radiation will represent an addition† of around 20% to that represented by the term $\rho v_i v_j$ and therefore will help to mitigate the decrease of sound output with decreasing ρ_J/ρ_0.

A balanced view of the existing data, and one reasonably consistent with the preceding argument, seems to be that some falling-off from proportionality to $\rho_0 U^8 A/a_0^5$ with increasing ρ_J/ρ_0 occurs, but that it is represented better by a factor ρ_J/ρ_0 than by $(\rho_J/\rho_0)^2$. This means that acoustic power output for $U/a_0 < 1.5$ is about $10^{-4}(\frac{1}{2}\rho_J U^3 A/a_0^5)$, in agreement with the law, stated several times in this paper, that acoustic efficiency is about $10^{-4}(U/a_0)^5$.

Appendix B: Simple-Source Theories of Jet Noise

Ribner[34] and others (cited by him) have described jet noise and other aerodynamic-sound phenomena in an alternative manner using simple sources. They define a function ϕ such that

$$\nabla^2\phi = \sum_{i=1}^{3}\sum_{j=1}^{3} \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \tag{B1}$$

in terms of which Eq. (8) becomes

$$\frac{\partial^2 \rho}{\partial t^2} - \sum_{i=1}^{3} a_0^2 \frac{\partial^2 \rho}{\partial x_i^2} = \nabla^2\phi \tag{B2}$$

This new equation is evidently the same as (8) but with T_{ij} replaced by the isotropic tensor $\varphi\delta_{ij}$. The solution [compare Eq. (9)] is

$$a_0^2(\rho - \rho_0) = \nabla^2 \int_V \frac{[\phi]}{4\pi r}\, d\tau = \frac{1}{a_0^2}\frac{\partial^2}{\partial t^2}\int_V \frac{[\phi]}{4\pi r}\, d\tau = \int_V \frac{[\ddot{\phi}/a_0^2]}{4\pi r}\, d\tau \tag{B3}$$

The equality of the second and third terms in (B3) follows from the fact that the retarded potential is a solution of the wave equation. The fourth term shows that the acoustic radiation is equivalent to that generated by a distribution of sources of strength $\ddot{\phi}/a_0^2$ per unit volume. It should be remarked that ϕ can be regarded as a sort of incompressible-flow approximation to the pressure.

Some mathematical criticisms of these theories in their earlier forms were made, particularly in relation to the treatment of source convection, but these criticisms appear to be met fully in Ribner's latest paper.[34] The correct directional factor [that is, for subsonic jets, $(1 - M_c \cos\theta)^{-5}$] is now deduced. Furthermore, the author proves conclusively that his formal expressions for the acoustic radiation field are equivalent mathematically to the theory set out in this lecture.

The relative value of these two mathematically equivalent formulations can accordingly be assessed only on the basis of which gives more help in estimating how the noise generated in various regions of a jet depends upon all the relevant variables, on the basis of approximations suggested by physical argument and experiments on jet turbulence. The simple-source theory has three main difficulties from this point of view:

1) One essential feature of T_{ij} is that it is small, of the order of the *square* of the disturbances, outside the jet itself (Sec. 2), so that the integral for the sound field that has to be estimated is merely an integral over the region of the jet. This is not true of ϕ or of the pressure (to which it represents an approximation), so that, in the simple-source theory, noise generation from outside the jet has, rather artificially, to be postulated and, presumably, estimated.

2) Inside turbulent jets extensive measurements of the correlation of velocity values at different points, and of how it falls away with increasing distance between the points, have been made. These, with further information on velocity spectra and space-time correlations, were used in the estimation (Secs. 3 and 4) of the sound generation associated with the quadrupole distribution T_{ij}, with reason, because its fluctuations arise mainly from velocity fluctuations. By contrast, little is known of pressure fluctuations inside turbulent jets, whose correlations and spectra cannot be measured directly. Estimation of fluctuations in ϕ though fluctuations in the pressure (to which it is some sort of approximation) is therefore not feasible, whereas direct estimation through Eq. (B1) defining ϕ leads right back to the problem of fluctuations in T_{ij} itself.

3) Furthermore, values of T_{ij} are expected to be well correlated only within convected eddies of limited extent and limited lifetime, and, for at any rate subsonic jets, the eddy size is very small compared with the wavelength of the sound that it generates, whose estimation is made much easier by this fact. By contrast, the correlation of φ at two points is expected to fall off much more slowly with the distance between them. Indeed, it appears doubtful whether correlation radii for ϕ are small enough compared with the wavelength, for any jet Mach numbers, to permit the neglect of differences between retarded times where values of ϕ are correlated significantly, for this neglect would lead to a uniform directional distribution of acoustic radiation (except for the convective factor), a serious error in every case when T_{ij} departs significantly from isotropy. It is, in fact, easy to show (one example is given in a footnote to the Bakerian Lecture) that a localized variation in T_{ij} produces a solution of Eq. (B1) for

† The author's 1954 paper,[13] on p. 22, estimated this proportion as $\frac{15}{4}$ of the square of the ratio of radian frequency to mean shear (a ratio now known, from Fig. 10, to be around 0.22).

φ whose correlation with that local variation falls off in general like r^{-3}. This makes impossible in simple-source theory a step like that from Eq. (11) to Eq. (12), whose validity depends on V_e being a volume integral of the relevant correlation, an integral that in this case diverges. Ribner points out correctly that the exact integral for the sound radiation field is not divergent, because for sound of each frequency it includes a sinusoidally varying term (with wavelength that of the sound itself), associated with differences in retarded times, which renders it convergent as the region of the integration becomes large compared with the wavelength. This means, however, that ordinary eddy-size measurements cannot be used in the estimation process, and also that differences in retarded times never can be neglected. It may be significant in the light of the foregoing remarks, that Mollö-Christensen's measurements[31] of pressures just outside a jet show (see his Fig. 12) extremely long tails to the correlation curve.

The three difficulties noted here seem to indicate that $\phi\delta_{ij}$ is not a useful alternative form of T_{ij}. There are not such severe difficulties with the $p_{ij} - a_0^2\rho\delta_{ij}$ term in T_{ij}, discussed briefly in Appendix A, because outside the jet itself its variations are negligible, and the large correlation spheres reduce, accordingly, to their intersections with the jet itself.

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